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ANOVA

ANCOVA

MULT REGR

- ANOVA and ANCOVA are special cases of the more general form of multiple regression
- We **model** the DV using a linear equation
- instead of modeling the DV using a weighted sum of continuous variables (X weighted by betas),
- we are modeling the DV using a series of **constants**
  - an overall constant mu
  - plus different constants alpha_j, one for each group
  - the least-squares estimates for constants are the means of each group

ANOVA $$Y_{ij} = \mu + \alpha_j + \epsilon_{ij}$$

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Repeated Measures Designs

- “within-subjects”
- each subject contributes a score for <u>each level of a factor</u>
- each subject contributes multiple scores
- subjects can serve as their own control
- variance between different conditions is no longer due to [effect + between-group sampling variance]
- it’s the same group of subjects! there is no “between-group” sampling variance
- variance only due to the effect

Examples

- effects of placebo, drug A and drug B can be studied in the same subjects; each subject can serve as their own control

- behaviour of subjects can be studied over time; a measurement can be taken from the same subjects at multiple time points

Advantages of Repeated Measures Designs

- more information is obtained from each subject than in a between-subjects design
    - within-subjects design: each subject contributes ***a*** scores (a is the number of conditions tested)
    - between-subjects design: each subject contributes only ***one*** score
    - # of subjects needed to reach a given level of statistical power is often much lower with within-subjects designs

Advantages of Repeated Measures Designs

- variability in individual differences between subjects is totally removed from the error term
- each subject serves as his/her own control
- error term is reduced
- statistical power increases

Analysis of Repeated Measures Designs

- 10 subjects
- each contributes 4 scores on DV
- one for each of 4 conditions
- **as an exercise**, let's treat this as a between-subjects design
- single-factor ANOVA

		Treatment Condition			
		1	*2*	*3*	*4*
Subject	*1*	8	10	7	5
	2	9	9	8	6
	3	7	5	8	4
	4	9	6	5	7
	5	8	7	7	6
	6	5	4	4	3
	7	7	6	5	4
	8	8	8	6	6
	9	9	8	6	5
	10	7	7	4	5

Source	SS	df	MS	F	sig
Factor	38.9	3	12.967	6.062	0.002
Error	77.0	36	2.139		
Total	115.9	39			

Analysis of Repeated Measures Designs

- what we are missing out on is the fact that some of the variance in the data is due to differences between **subjects**
- what if we were to include a second factor, namely "subjects"?
- We don't have enough df for both main effects + the interaction Subjects x Factor
- So we will limit the model to:
  - main effect of Factor
  - main effect of Subjects

		Treatment Condition			
		1	*2*	*3*	*4*
	1	8	10	7	5
	2	9	9	8	6
	3	7	5	8	4
	4	9	6	5	7
Subject	*5*	8	7	7	6
	6	5	4	4	3
	7	7	6	5	4
	8	8	8	6	6
	9	9	8	6	5
	10	7	7	4	5

Analysis of Repeated Measures Designs

- now we have reduced the error term by accounting for another portion of the variance
- variance due to differences among subjects

Source	SS	df	MS	F	sig
Factor	38.9	3	12.967	12.241	0.000…
Subjects	48.4	9			
Total	28.6	27	1.059		
	115.9	39			

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Repeated Measures ANOVA

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	115.9	39			

Competing Models

full model $Y_{ij} = \mu + \alpha_j + \pi_i + \epsilon_{ij}$

restricted model $Y_{ij} = \mu + \pi_i + \epsilon_{ij}$

- full model includes effect of factor and effect of subjects
- restricted model only includes effect of subjects (effect of factor is zero)
- so the difference here compared to regular "between-subjects" models is simply the inclusion of terms accounting for the effects of subjects
- remember: the more variance you can account for, the smaller the error term, the higher the F value, and the more powerful the statistical test

Analysis of Repeated Measures Designs

- just as always, we can compute an F statistic based on Error for the full model and Error for the restricted model

$$F = \frac{(E_R - E_F)/(df_R - df_F)}{E_F/df_F}$$

$$df_F = (n-1)(a-1)$$

$$df_R - df_F = (a-1)$$

- see Chapter 11 for all the gory details

Assumptions

- random sampling from population
- independence of subjects
- normality
- homogeneity of treatment-difference variances
  - variance of difference scores between any two levels of a factor must be equal to variance of differences scores between all other pairs of levels of the factor
  - equivalent to showing that the population covariance matrix has a certain form, that is, it displays the property of ***sphericity***
  - this is all very mathematical and we don't need to know the details
  - fortunately there is (1) a test to see if we have violated the assumption, and (2) a method to correct for violations

Homogeneity of Treatment-Difference Variances

- We will see how to perform a test of sphericity in R
- R will report a number of corrected versions of the F test assuming sphericity is violated
- "Greenhouse-Geisser" adjustment adjusts the degrees of freedom (reducing them) so that Fcrit is larger (more conservative test)
- many people use G-G
- others like Huynh-Feldt because it's slightly less conservative

Comparisons Among Individual Means

- we can use the same formulas we used in between-subjects designs to test any contrast:

$$F = \frac{SS_\psi}{MSErr}$$

$$SS_\psi = \frac{n(\psi)^2}{\sum c_j^2}$$

- caveat: tests of comparisons among means are very sensitive to violations of the sphericity assumption
- methods exist to circumvent this by using different error terms (see Chapter)

Experimental Design Considerations

- Order Effects
  - e.g. a neuroscientist wants to compare the effects of Drug A and Drug B on aggressiveness in pairs of monkeys
  - every pair of monkeys will be observed under the influence of both Drug A and Drug B
  - How should we conduct the study?
  - one possibility: administer Drug A to every pair, observe the subsequent interactions, and then administer Drug B to every pair
  - bad idea: confounds potential drug differences with the possible effects of time
  - even if a significant difference between the drugs is obtained, it may not have occurred because the drugs truly have a different effect
  - it may be because monkeys were simply becoming less aggressive over time
  - or: a significant drug difference could be missed because of time effects

Counterbalancing

- a solution is to counter-balance the order in which treatments are administered
- e.g. Drug A then Drug B to half the monkeys;
- Drug B then Drug A to the other half
- monkeys are randomly assigned to each group
- known as a "crossover design"

Differential Carryover Effects

- a nasty potential problem
- occurs when the carryover effect of treatment condition 1 onto treatment condition 2 is **different** than the carryover effect of treatment 2 onto treatment condition 1
- counterbalancing will NOT control for this problem
- one solution is a "washout period" after the administration of one treatment, to let enough time elapse so that the next treatment is no longer affected
- can't always be done: some carryover effects are permanent (e.g. learning, memory, lesions, etc)
- some scientific questions are better suited to between-subjects designs

Counterbalancing more than two levels

- what if we want to counterbalance an experiment with more than two levels? (e.g. 4)
- there are actually 24 different orderings of 4 conditions
- we would need 24 subjects to represent each order only once!
- Two alternatives:
  - randomize the order for each subject; order effects will be controlled for "in the long run"
- Latin Square Designs
  - an arrangement of conditions so that each condition appears exactly once in each possible order

Latin Square Designs

Order				
Ss	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Advantages of Repeated Measures Designs

- each subjects contributes a x n data points; fewer subjects are required
- increased power to detect true treatment effects due to a smaller error term

Disadvantages of Repeated Measures Designs

- risk of differential carryover effects
- within vs between subjects designs may not be addressing the same conceptual question even though the manipulated variables appear to be the same
- In a within-subjects design every subject experiences each treatment *in the context of all other treatments*
- In a between-subjects design every subject only ever experiences a single treatment, in isolation
- simply a different situation

Two Factor Repeated Measures

- each subject contributes a score on the DV for **every** level of **both** factors
- e.g. Factor A (2); Factor B (3)

Factor A	A1			A2		
Factor B	B1	B2	B3	B1	B2	B3
Subject 1	420	420	480	480	600	780
Subject 2	480	480	540	660	780	780
Subject 3	540	660	540	480	660	720
Subject 4	480	480	600	360	720	840
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- note something that distinguishes a repeated measures design from a between-subjects design:
- there is no "within cell" variance
- there is only a single # for each condition per subject
- variance within a condition (e.g. A1B1) exists only due to the fact that there are scores from different subjects
- this affects the computation of the error term in the ANOVA
- error term is no longer simply "within-cell" variance
- error terms are effects "within subjects"

Two Factor Repeated Measures

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- Issues of analysis are identical to a between-subjects design
- we are interested in testing:
    - A main effect
    - B main effect
    - A x B interaction effect
    - and any follow-up tests of individual means
- what is different is simply the calculation of the error term(s)
- and **which error terms** are used for testing **which effect**

GLM

- lets assume (like last week) that "subjects" is included as a factor in our model

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- now we have A, B, and S

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_i + (\alpha\beta)_{jk} + (\alpha\pi)_{ji} + (\beta\pi)_{ki} + (\alpha\beta\pi)_{jki} + \epsilon_{ijk}$$

- main effects: A, B, S
- 2-way interactions: AxB, AxS, BxS
- 3-way interaction: AxBxS

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AxS
BxS
AxB xS
are error terms

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Source	SS	df	MS	F	sig
S		4			
A		1			
A x S		4			
B		2			
B x S		8			
A x B		2			
A x B x S		8			

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a different error term for testing each effect

Source	SS	df	MS	F	sig
S		4			
A		1			
A x S		4			
B		2			
B x S		8			
A x B		2			
A x B x S		8			

Different Error Terms

- different error terms are used for the F-test for each different effect
- thus the total error is split into three error terms
- this helps us - we get smaller error terms
- therefore larger F values
- more powerful statistical test

Source	SS	df	MS	F	sig
S		4			
A		1			
A x S		4			
B		2			
B x S		8			
A x B		2			
A x B x S		8			

Meaning of Error Terms

- Error terms here are **interaction terms** between an "effect" (e.g. A or B or A x B) and subjects (S)
- remember the meaning of an interaction
  - effect in question differs across levels of the other factor
- e.g. A x S means that effect of factor A is different across different subjects
- A x S therefore captures variance of the "A" effect across different subjects - this is the appropriate error term (denominator of F test for the "A" effect)

Source	SS	df	MS	F	sig
S		4			
A		1			
A x S		4			
B		2			
B x S		8			
A x B		2			
A x B x S		8			

- Table 12.5, Chapter 12 M&D

Source	SS	df	MS	F	sig
S	33600	4			
A	147000	1	147000	17.5	0.014
A x S	33600	4	8400		
B	138480	2	69240	14.16	0.002
B x S	39120	8	4890		
A x B	67920	2	33960	11.67	0.004
A x B x S	23280	8	2910		

- 3 different F-tests, 3 different error terms

★ when conducting follow-up tests between individual means, you need to use the appropriate error term

Follow-Up Tests - Which Error Term?

$$F = \frac{SS_{\psi}}{MSErr}$$

$$SS_{\psi} = \frac{\tilde{n}(\psi)^2}{\sum c^2}$$

$\tilde{n} =$ # Ss in each mean

Source	SS	df	MS	F	sig
S	33600	4			
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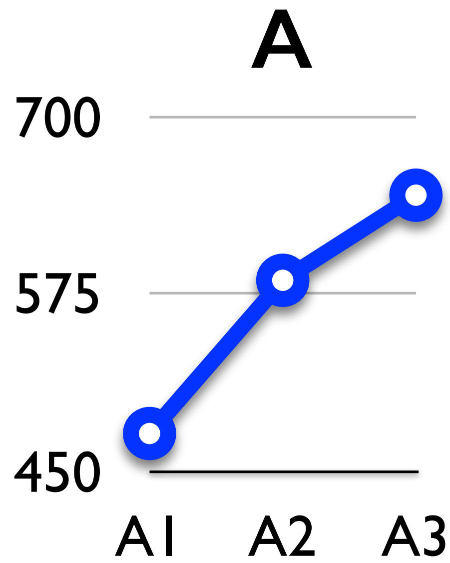

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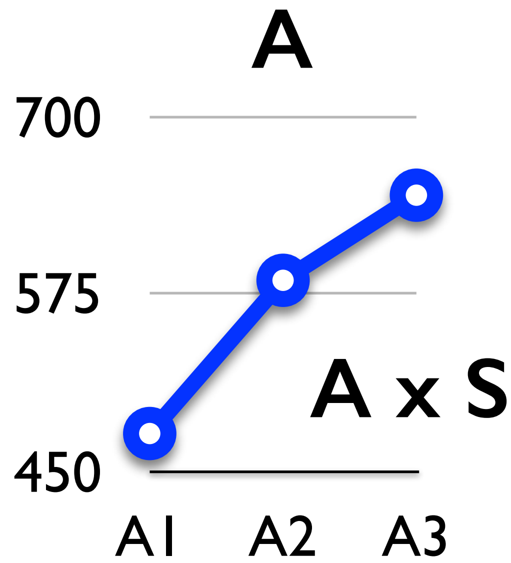

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A x S	33600	4	8400		
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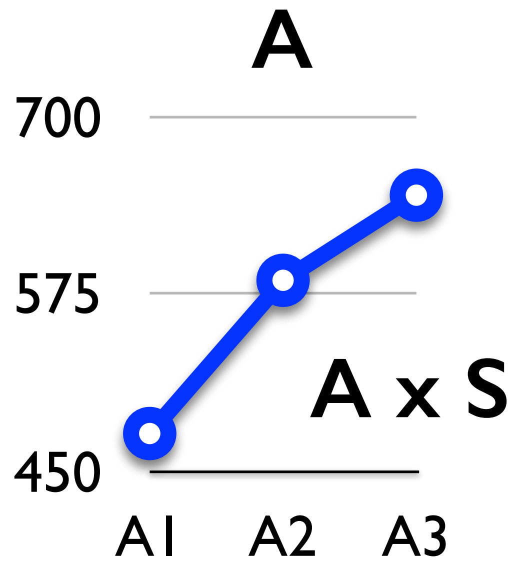

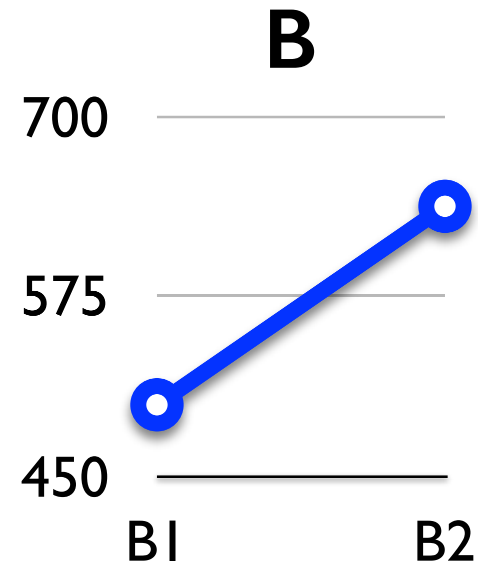

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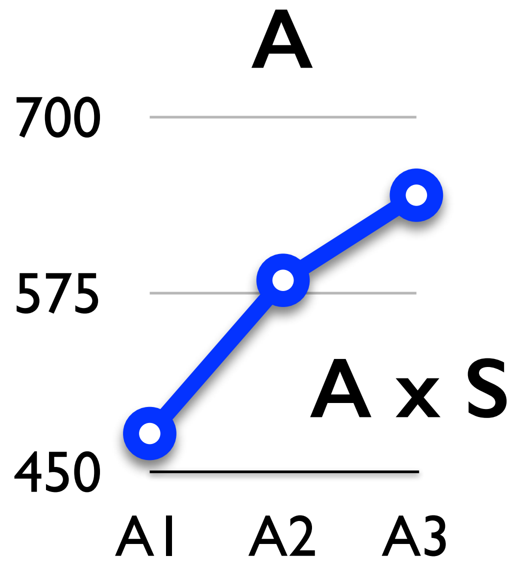

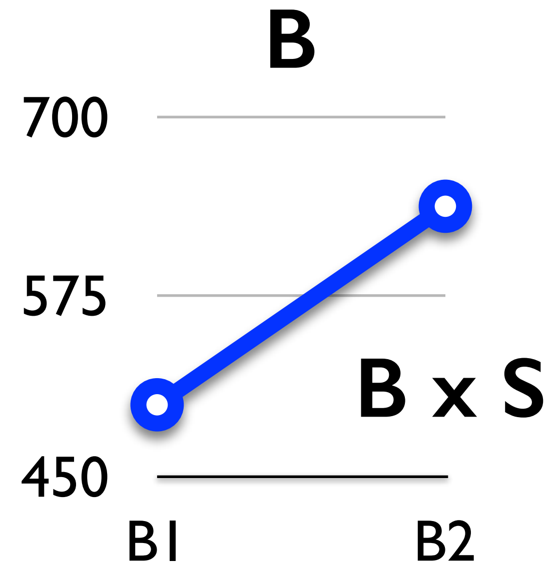

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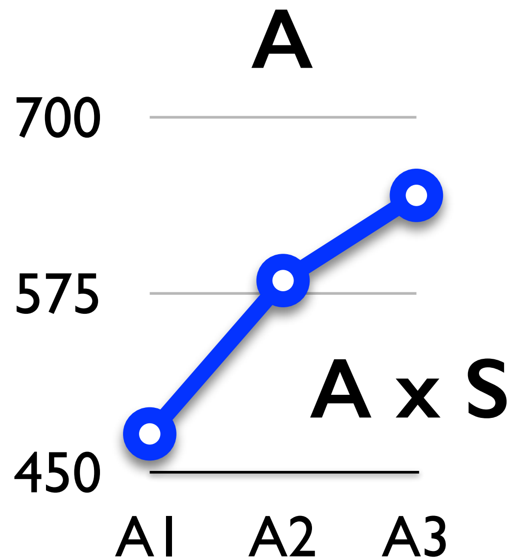

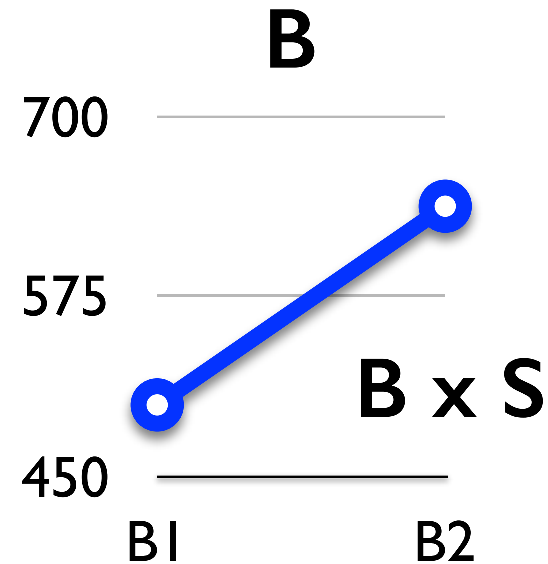

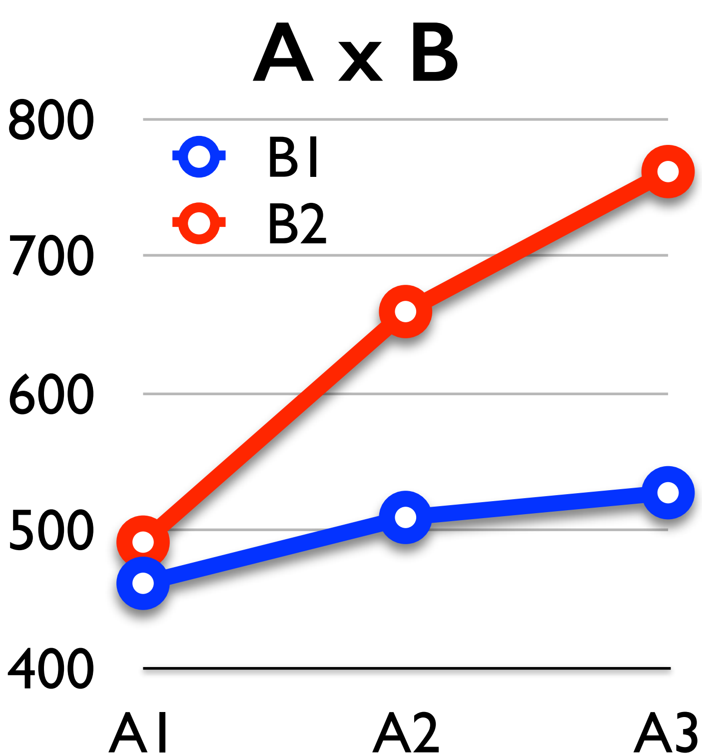

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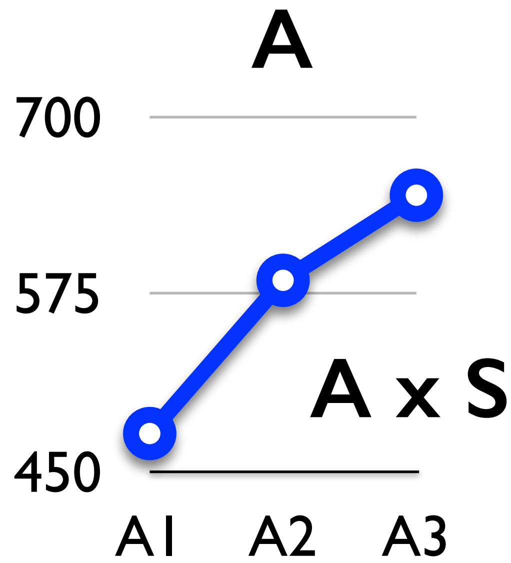

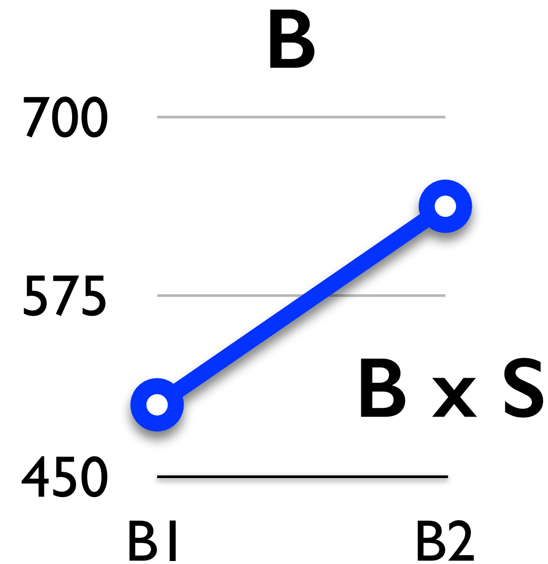

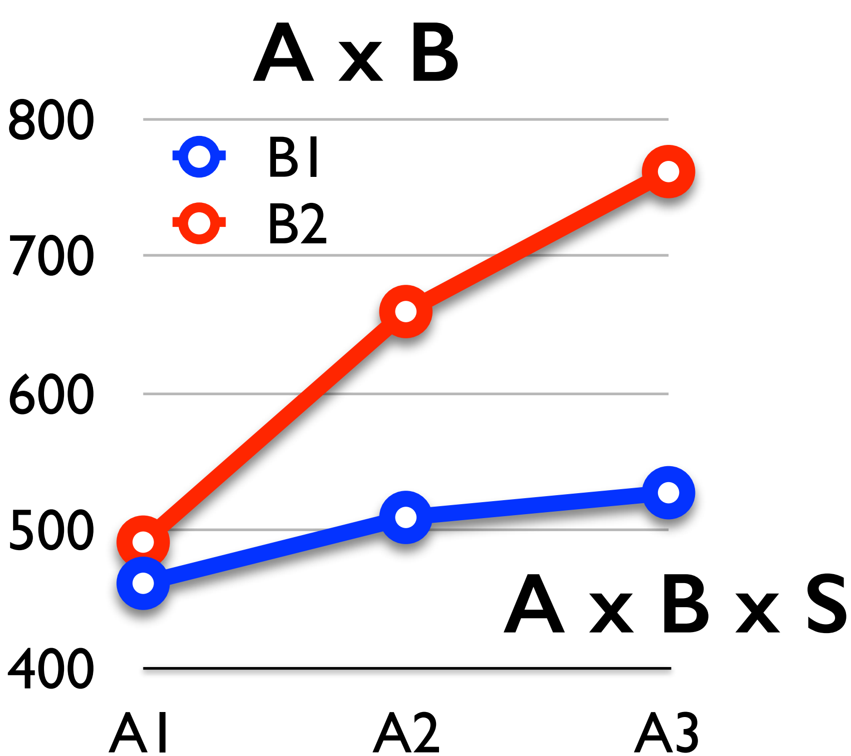

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Separate vs Pooled (the same) Error Terms

- when homogeneity of variance assumption is violated, a separate error term can be computed for each different contrast
- otherwise the appropriate error term from the ANOVA table can be used
- these are called "pooled error terms"
- See Chapter 12 for details of separate error term calculation

Mixed (Split-Plot) Designs

- one between-subjects factor, and
  one within-subjects factor
- naturally suited to studying different groups of subjects over time
  - group is between-subject factor
  - time is within-subject factor
- sometimes called a "split-plot" design
  - a historical holdover from its uses in agricultural research

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
Sub3	5.6	4.1	1
Sub4	4.3	6.4	2
Sub5	6.6	7.7	2
Sub6	7.8	8.2	2

GLM

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
Sub3	5.6	4.1	1
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Sub5	6.6	7.7	2
Sub6	7.8	8.2	2

- Factor A is between-subjects
- Factor B is within-subjects

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_{i(j)} + (\alpha\beta)_{jk} + (\beta\pi)_{ki(j)} + \epsilon_{ijk}$$

- subjects (pi) appears in only two terms now
  - main effect of subjects
  - interaction with B (repeated measures effect)
- no interaction with A - subjects are not crossed with A
  - each subjects only provides a score in one (not all) levels of A

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
Sub3	5.6	4.1	1
Sub4	4.3	6.4	2
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Source	

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
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Source	explanation

Choice of Error Term

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Sub1	2.3	3.4	1
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$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_{i(j)} + (\alpha\beta)_{jk} + (\beta\pi)_{ki(j)} + \epsilon_{ijk}$$

Source	explanation
A	

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
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$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_{i(j)} + (\alpha\beta)_{jk} + (\beta\pi)_{ki(j)} + \epsilon_{ijk}$$

Source	explanation
A	main effect of Factor A

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
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Source	explanation
A	main effect of Factor A
S/A	

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
Sub3	5.6	4.1	1
Sub4	4.3	6.4	2
Sub5	6.6	7.7	2
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or "S/A" = variance due to

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
Sub2	3.3	5.2	1
Sub3	5.6	4.1	1
Sub4	4.3	6.4	2
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or "S/A" = variance due to
B	

Choice of Error Term

	B1	B2	A
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or "S/A" = variance due to
B	main effect of Factor B

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or “S/A” = variance due to
B	main effect of Factor B
A x B	

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or "S/A" = variance due to
B	main effect of Factor B
A x B	interaction effect A x B

Choice of Error Term

	B1	B2	A
Sub1	2.3	3.4	1
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$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_{i(j)} + (\alpha\beta)_{jk} + (\beta\pi)_{ki(j)} + \epsilon_{ijk}$$

Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or “S/A” = variance due to
B	main effect of Factor B
A x B	interaction effect A x B
B x S/A	

Choice of Error Term

	B1	B2	A
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Source	explanation
A	main effect of Factor A
S/A	**Subjects error term S:** or "S/A" = variance due to
B	main effect of Factor B
A x B	interaction effect A x B
B x S/A	**error term is B x S:** or "B x S/A" :interaction of B

Split Plot

Split Plot

- everything else is the same as before

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- just like before, we can perform followup tests of individual means using an F test of a contrast

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Split Plot

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- just like before, we choose the appropriate error term as the denominator
- just like before, we compare compute p based on Fobs
- just like before, there are assumptions of homogeneity of variance & sphericity, and corrections if they are violated (e.g. G-G)