Assignment 9

Scientific Computing with MATLAB
due: Dec 6, 2018

1. Mass-Spring simulation

1. What is the influence of the sign and magnitude of the stiffness parameter k?
2. Damping can be used to reduce the magnitude of oscillations. Damping generates a force that is directly proportional to velocity ($F = -b\dot{x}$). Add damping to the mass-spring system and re-run the simulation. Specify the value of the damping constant $b = -2.0$. What happens?
3. What is the influence of the sign and magnitude of the damping coefficient b?
4. Double the mass, and re-run the simulation. What happens?

2. Lotka-Volterra Predator-Prey Model

1. Equations 2 and 3 in the "Integrating ODEs & simulating dynamical systems" course notes document give the Lotka-Volterra equations which can be used to model populations of predators and prey over time. The variable x is the number of prey (for example rabbits), and y is the number of predators (e.g. foxes). Assumptions of the model are:
   - prey find ample food at all times
   - food supply of predators depends entirely on prey population
   - the rate of change of the population is proportional to its size
   - the environment does not change

   The values of α, β, γ and σ are constants that characterize different aspects of the two populations. The parameters can be interpreted as:
   - α is the natural growth rate of prey in the absence of predation
   - β is the death rate per encounter of prey due to predation
   - σ is related to the growth rate of predators
   - γ is the natural death rate of predators in the absence of food (prey)

   Your task is to simulate this system. Start with the following values for the initial states, and the constants:
   - $(x_0, y_0) = (0.5, 0.5)$
   - $\alpha = 0.1$, $\beta = 0.1$, $\sigma = 0.1$, $\gamma = 0.1$
   - set up a time vector from 0 to 500 in steps of 1
2. Plot the population size of predator and prey over time
3. Plot the trajectory of the system in *state-space*, in other words plot $x(t)$ vs $y(t)$.
4. Increase the α constant to 0.2 and re-run. What happens?

5. Set all constants to 0.20 and re-run. What happens?
6. Try the following: $(\alpha, \beta, \sigma, \gamma) = (0.2, 0.2, 0.2, 0.0)$. What happens and why?

Please submit your MATLAB script to OWL.