

Assignment 2

Neuroscience 9520
Computational Models in Neuroscience
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Due Sep 30 2012 11:59pm EST

Submit your assignment as a single .pdf file sent by email to paul@gribblelab.org with the subject line `compneuro assignment 2`. Please name the pdf file with your last name and the assignment number, for example `gribble2.pdf`.

1 Modelling the Dynamics of Baseball Flight

The equations of motion governing the dynamics of the flight of a baseball traveling through air (in 2D) are the following:

$$\ddot{x} = (-k/m)(\dot{x}) \left(\sqrt{\dot{x}^2 + \dot{y}^2} \right) \quad (1)$$

$$\ddot{y} = (-k/m)(\dot{y}) \left(\sqrt{\dot{x}^2 + \dot{y}^2} \right) - g \quad (2)$$

where x and y are the horizontal and vertical position of the ball (metres), respectively, m is the mass of the ball (kg), g is the gravitational force (N), and k is a frictional coefficient that depends on the density of air, the surface area of the object, and the object's surface material (kg/m).

Assume the following constant values:

$$\begin{aligned} k & 5.9 \times 10^{-4} \text{ kg m}^{-1} \\ m & 0.15 \text{ kg} \\ g & 9.81 \text{ m s}^{-2} \end{aligned}$$

Questions

1. What is the order of the system?
2. Write a function in python that returns the state derivatives of the system given the current state. The header should be: `def baseball(state, t):`
3. Simulate 20 seconds of the system, in steps of 10 ms, starting from an initial position of $(x, y) = (0, 0)$ and an initial velocity of $(\dot{x}, \dot{y}) = (30, 50)$. Plot x and y position over time. Find the time, and x position, when the ball hits the ground ($y = 0$). Don't worry about explicitly modelling the ground, per se ... you can allow the ball to continue to fall into negative values of y . Just find the time at which the ball crosses the $y = 0$ line.
4. You can express initial velocity as (\dot{x}, \dot{y}) or alternatively as an elevation (angle above the horizon) and velocity (α, v) . They are related by the following equations:

$$\dot{x} = \cos(\alpha)v \quad (3)$$

$$\dot{y} = \sin(\alpha)v \quad (4)$$

Assume the initial position of the ball is $(x = 0, y = 0)$. Find the initial conditions for velocity (α, v) that will result in the ball hitting the ground ($y = 0$) 100 metres from the start ($x = 100$).

Include in your report the python code you used and any figures to illustrate your answers.