Signals, Sampling & Filtering

Psychology 9040
Scientific Computing

- Representation of Signals in Time vs Frequency Domain
- Fast Fourier Transform (FFT)
- Sampling
- Spectrum
- Filtering
- Quantization

Apple Inc.

NASDAQ: AAPL

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Overview | News | Compare | Financials

207.25 USD **–1.97 (0.94%)** ↓

Nov. 8, 11:35 a.m. EST · Disclaimer

1 day | 5 days | 1 month | 6 months | YTD | 1 year | 5 years | Max

Open	209.98	Div yield	1.41%
High	210.12	Prev close	209.22
Low	207.14	52-wk high	233.47
Mkt cap	983.72B	52-wk low	150.24
P/E ratio	17.47		

Time Domain Representation

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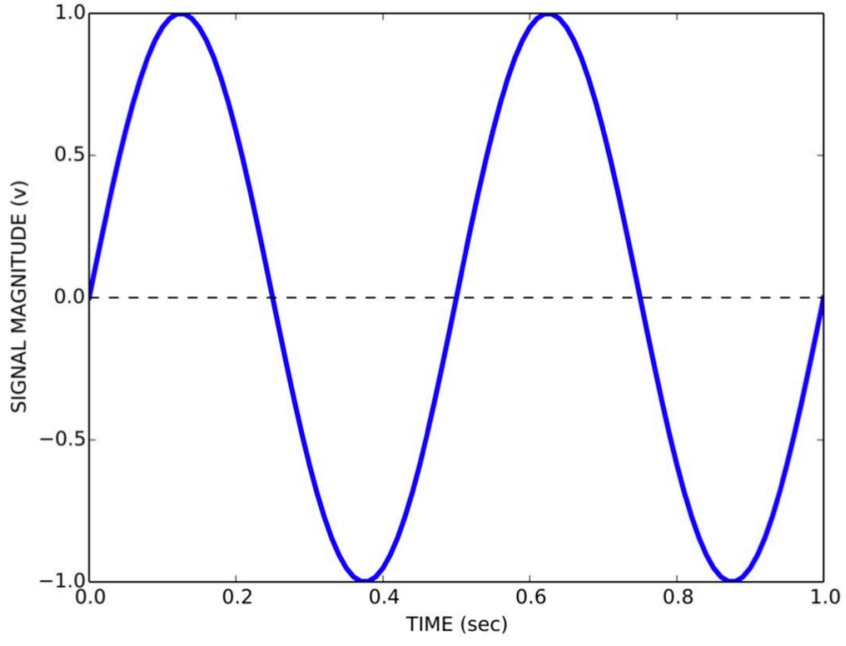
$$s(t) = \left(\frac{b}{2}\right)\sin(wt)$$

$$w = \frac{2\pi}{T}$$



Frequency Domain Representation

Apple Inc.

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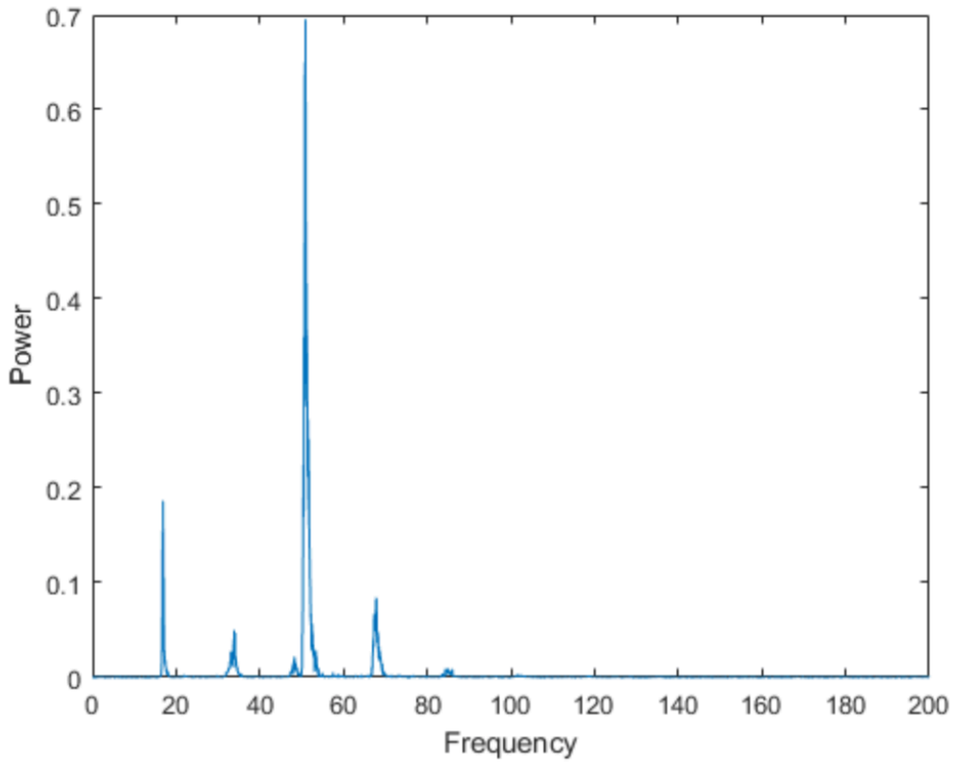
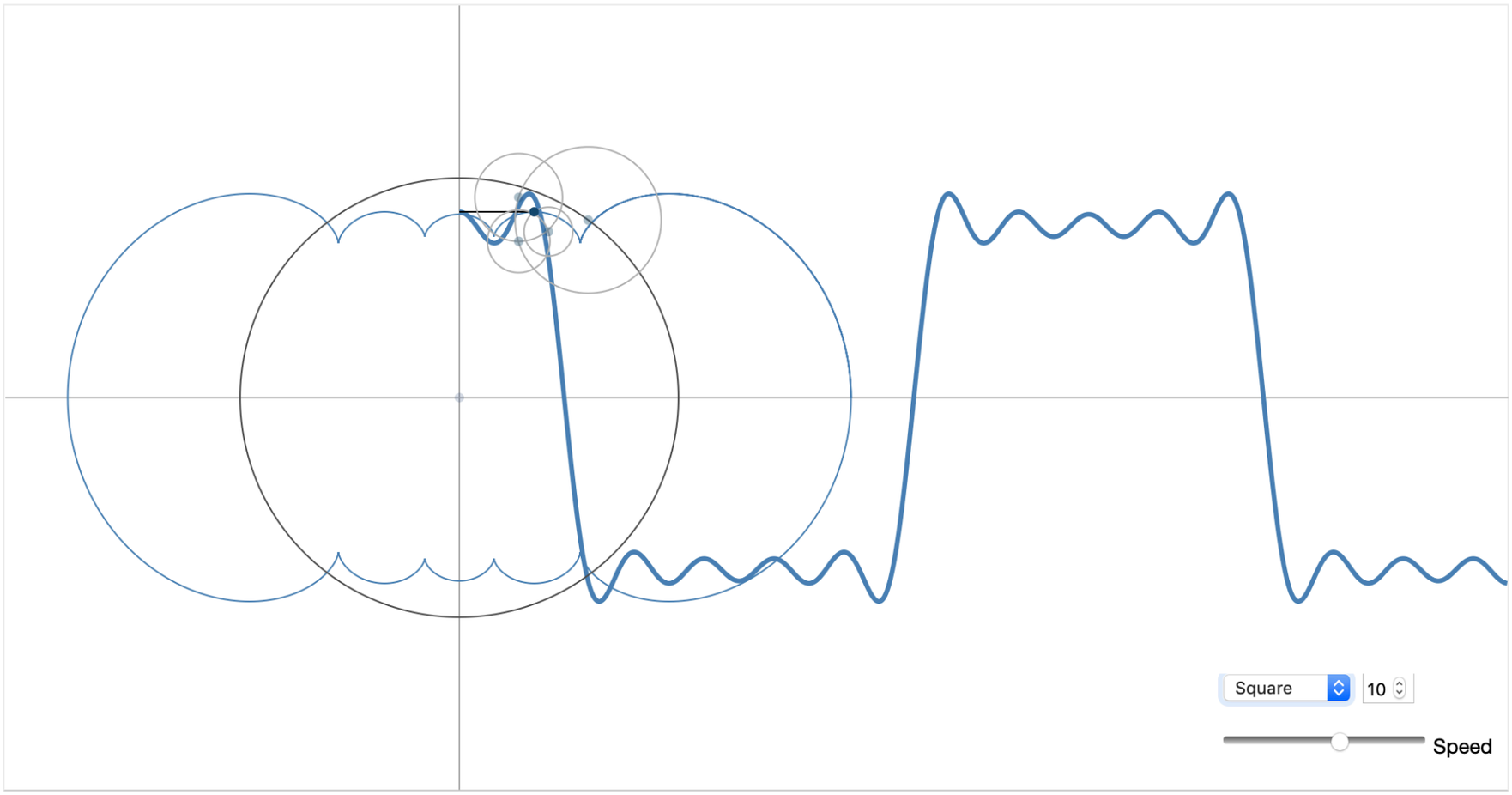
$$s(t) = \frac{a_0}{2} \sum_{n=1}^{\infty} [r_n \cos(nwt - \phi_n)]$$



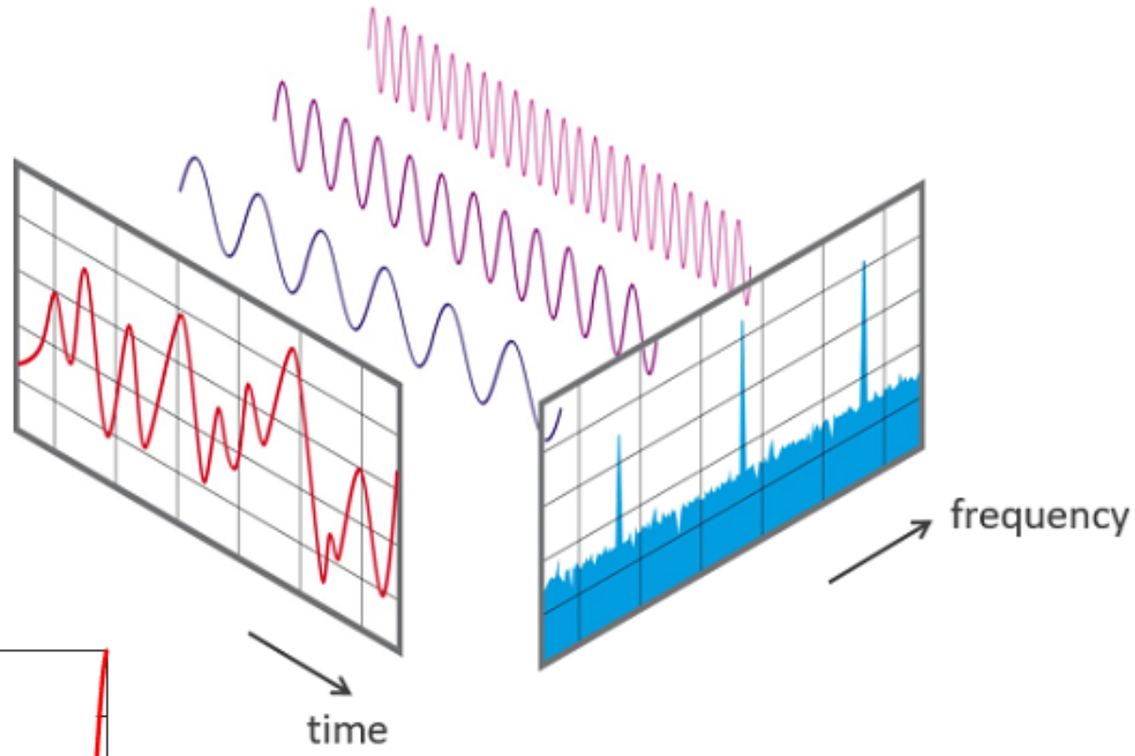
Fourier series visualisation

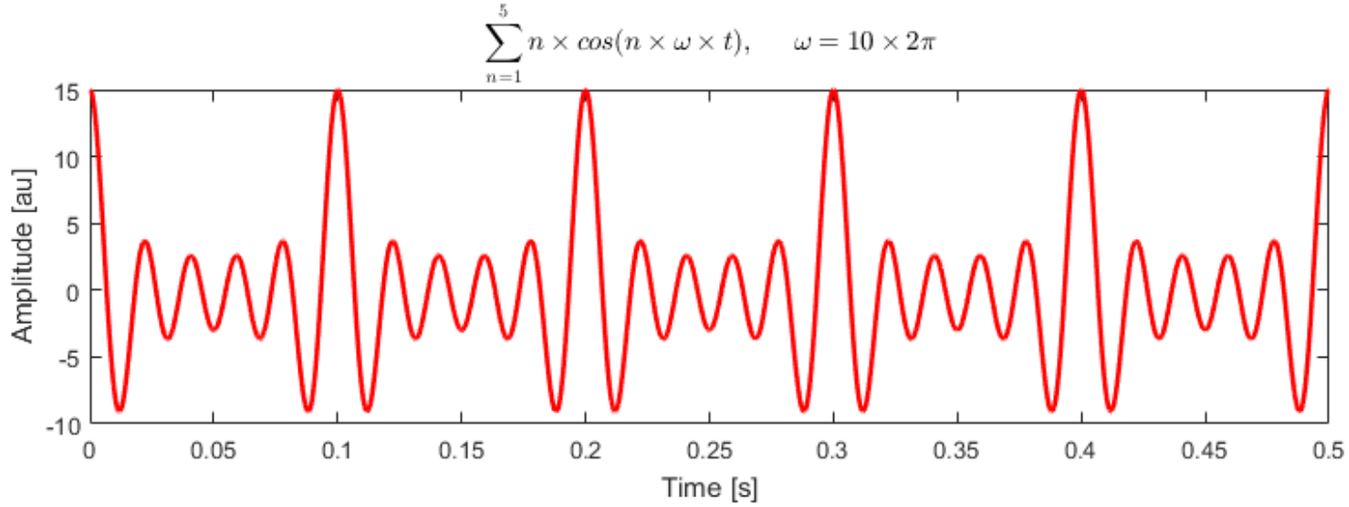
http://www.gribblelab.org/9040/fourier.html

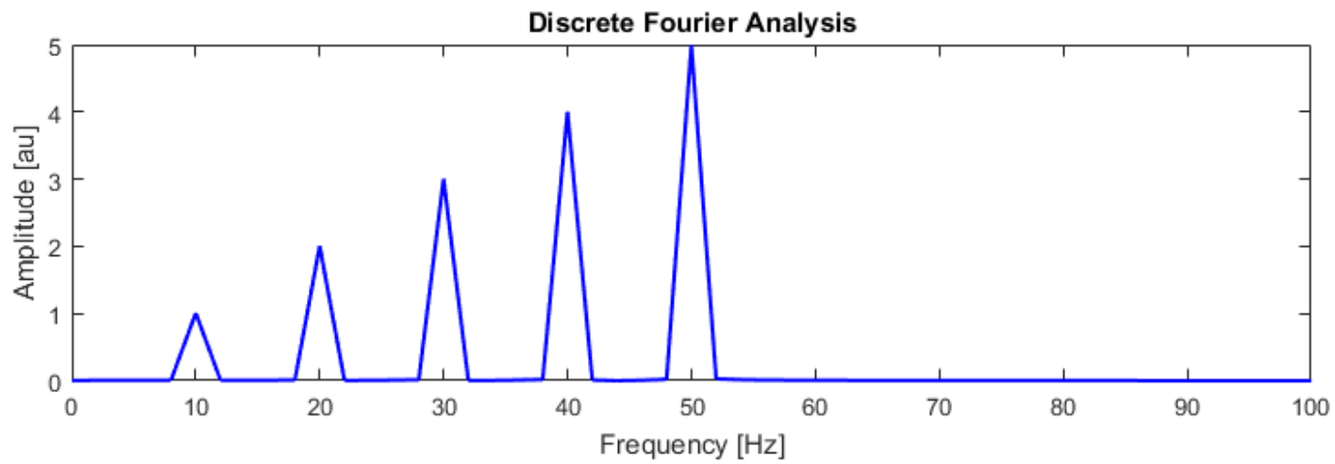


Fast Fourier Transform (FFT) Algorithm

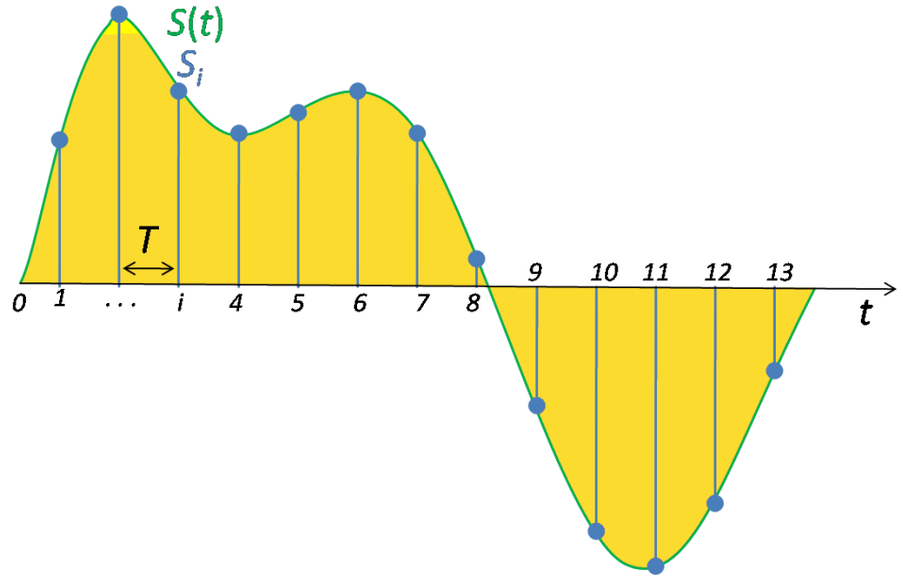
- MATLAB `fft()`
- Python `scipy.fft()` & `numpy.fft()`







Sampling



- Taking measurements of a continuous (e.g. analog) signal at discrete points in time

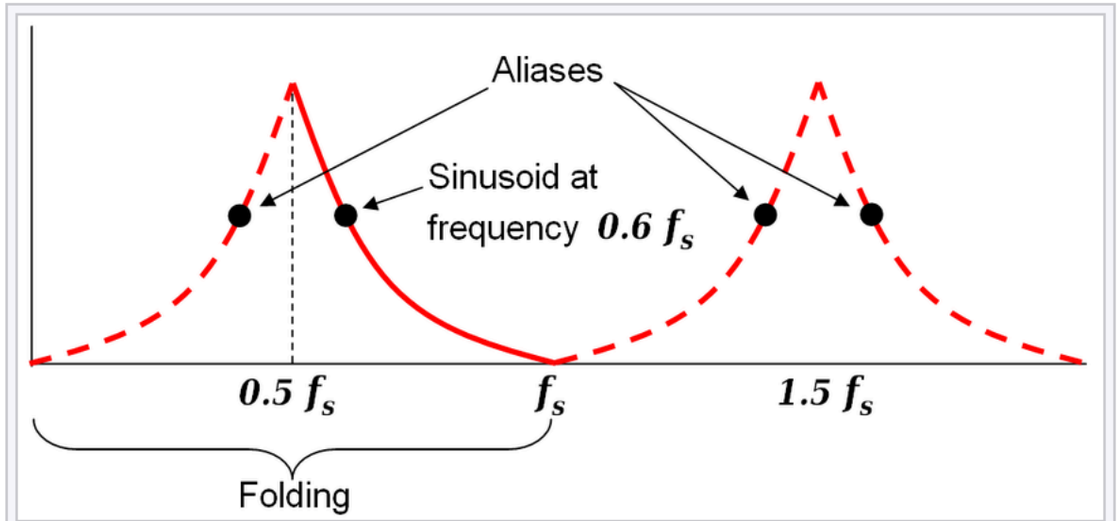
- Sampling rate, e.g. 1000 Hz

- Nyquist–Shannon sampling theorem & the Nyquist frequency

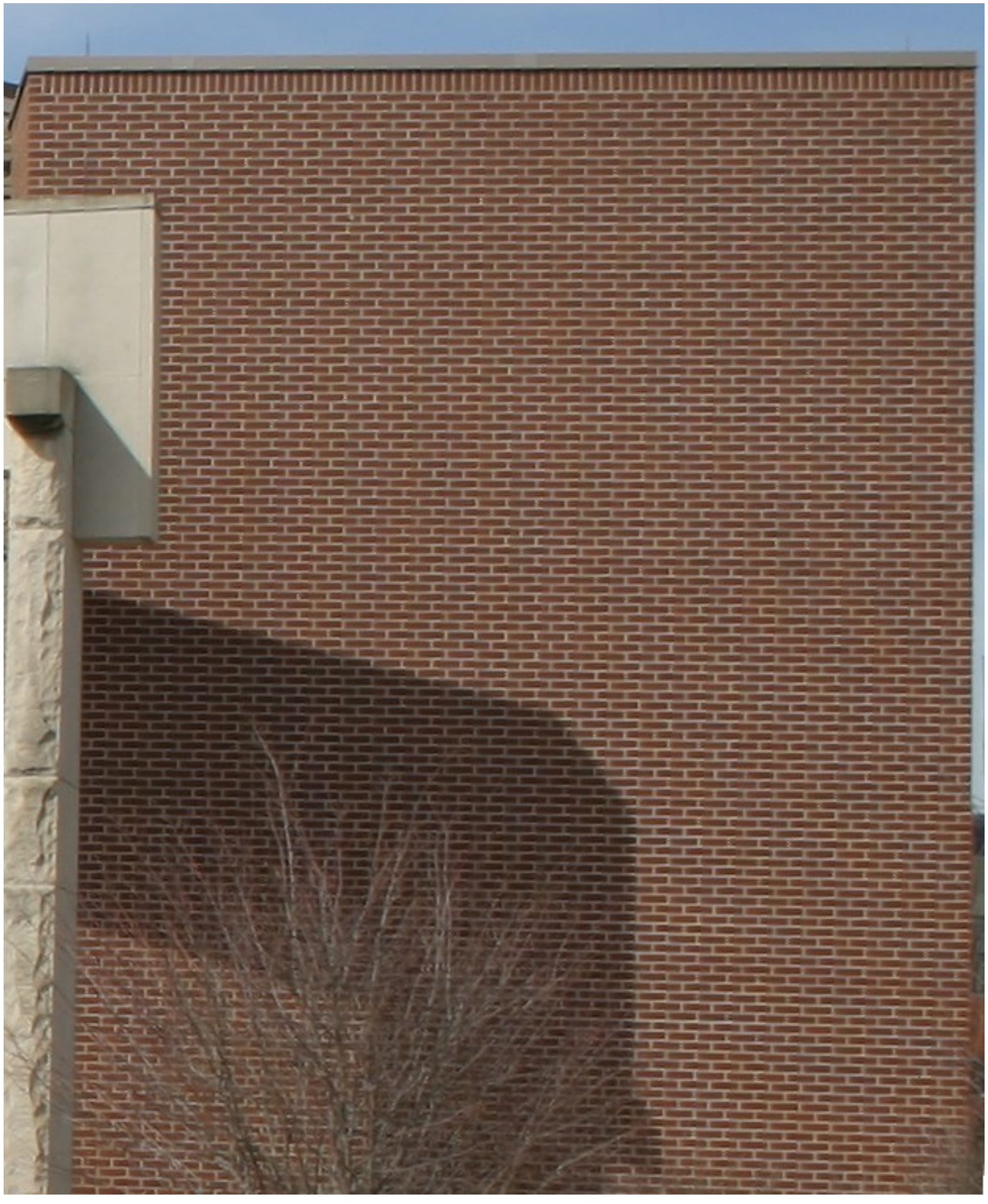
- aliasing

Aliasing

- Signal power in frequencies above the Nyquist frequency are aliased down into lower frequencies



The black dots are aliases of each other. The solid red line is an example of amplitude varying with frequency. The dashed red lines are the corresponding paths of the aliases.

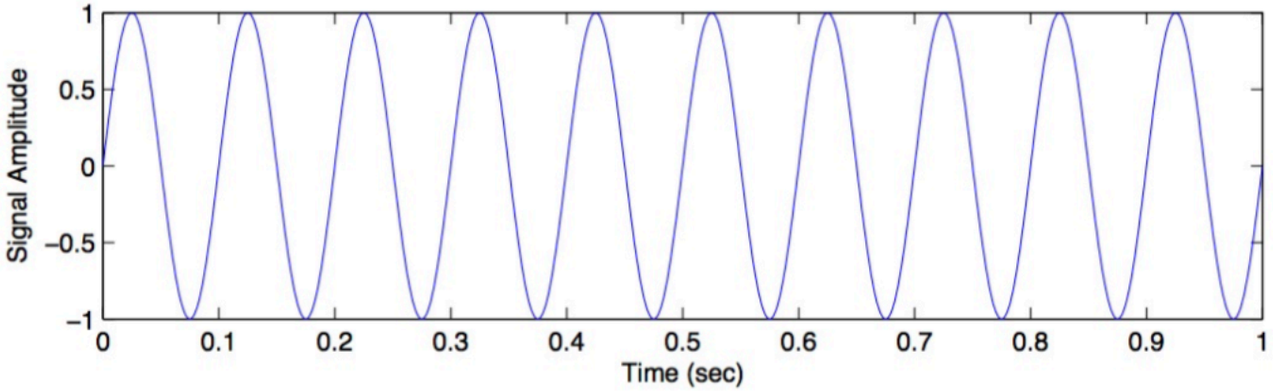


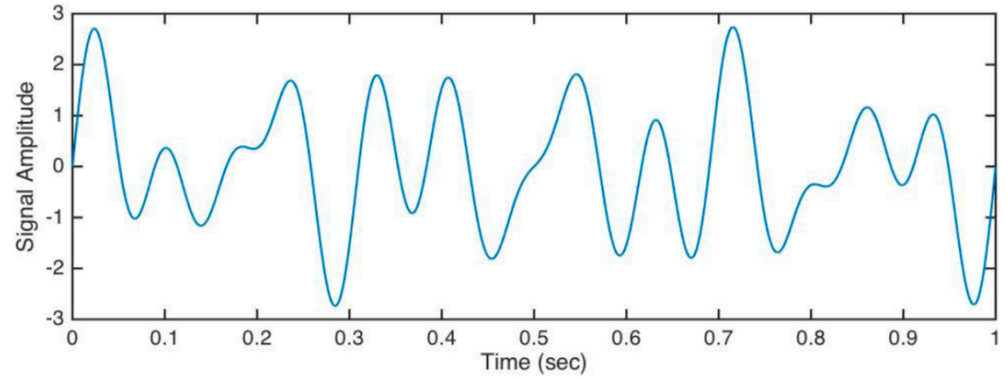
Downsampled without first low-pass filtering

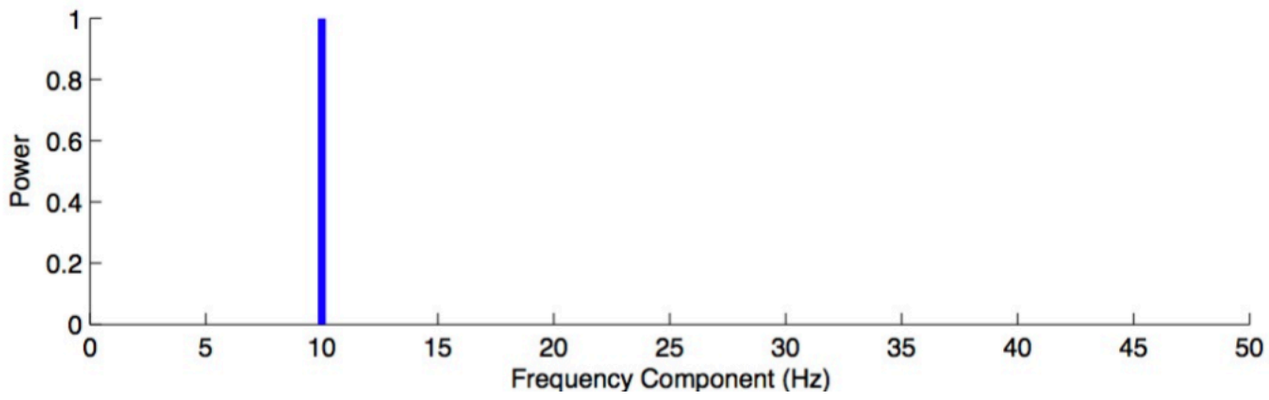
Moiré pattern

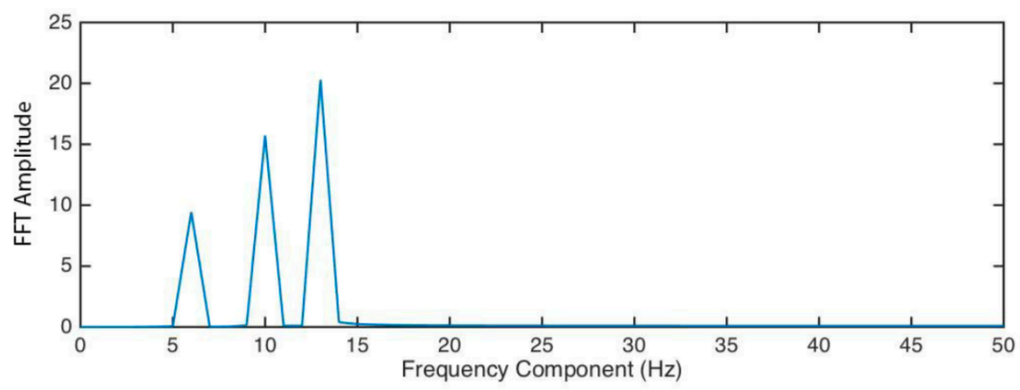
Spectrum

- Signal power at different frequencies



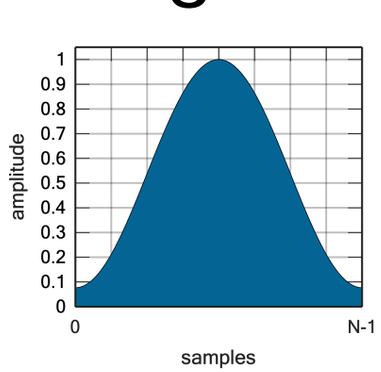




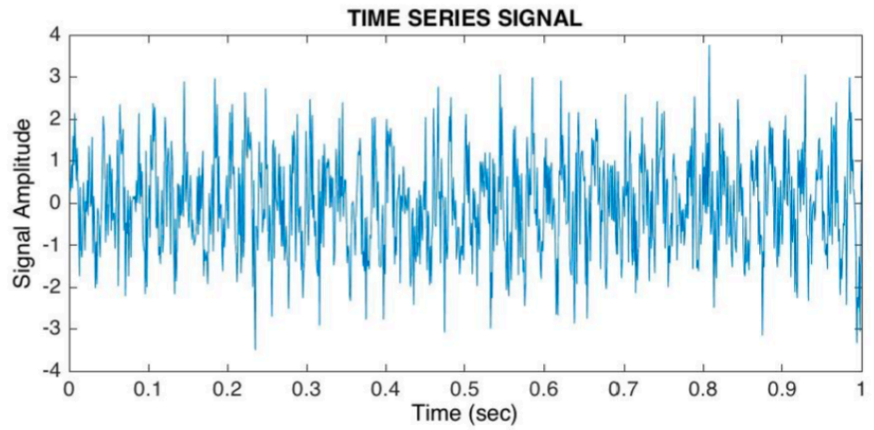


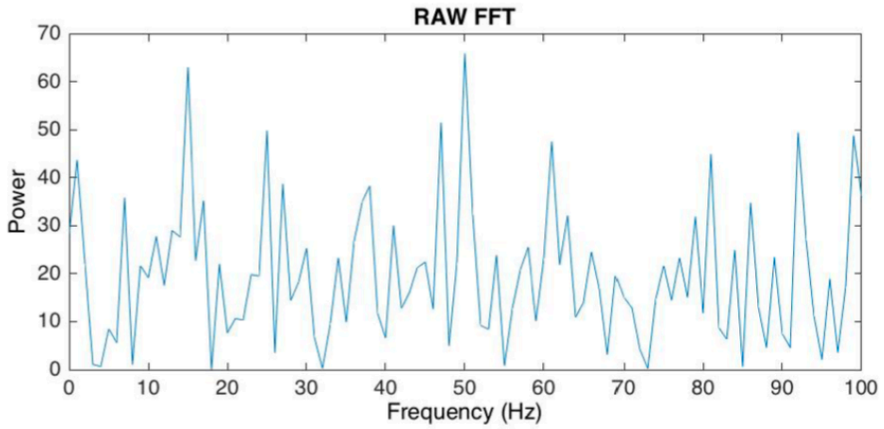
Power Spectral Density (PSD)

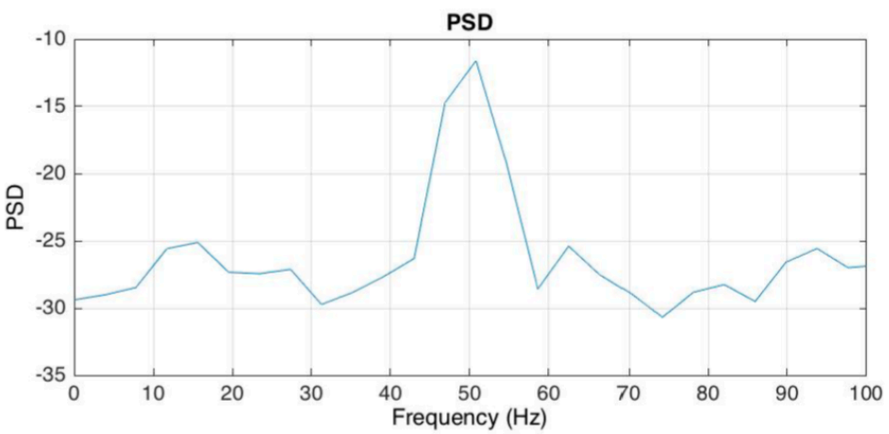
- Split signal into overlapping time windows
- Weight each window e.g. using a Hamming window



- FFT each window
- Average all the FFTs to get the PSD estimate







Spectrogram

- Spectrum over time

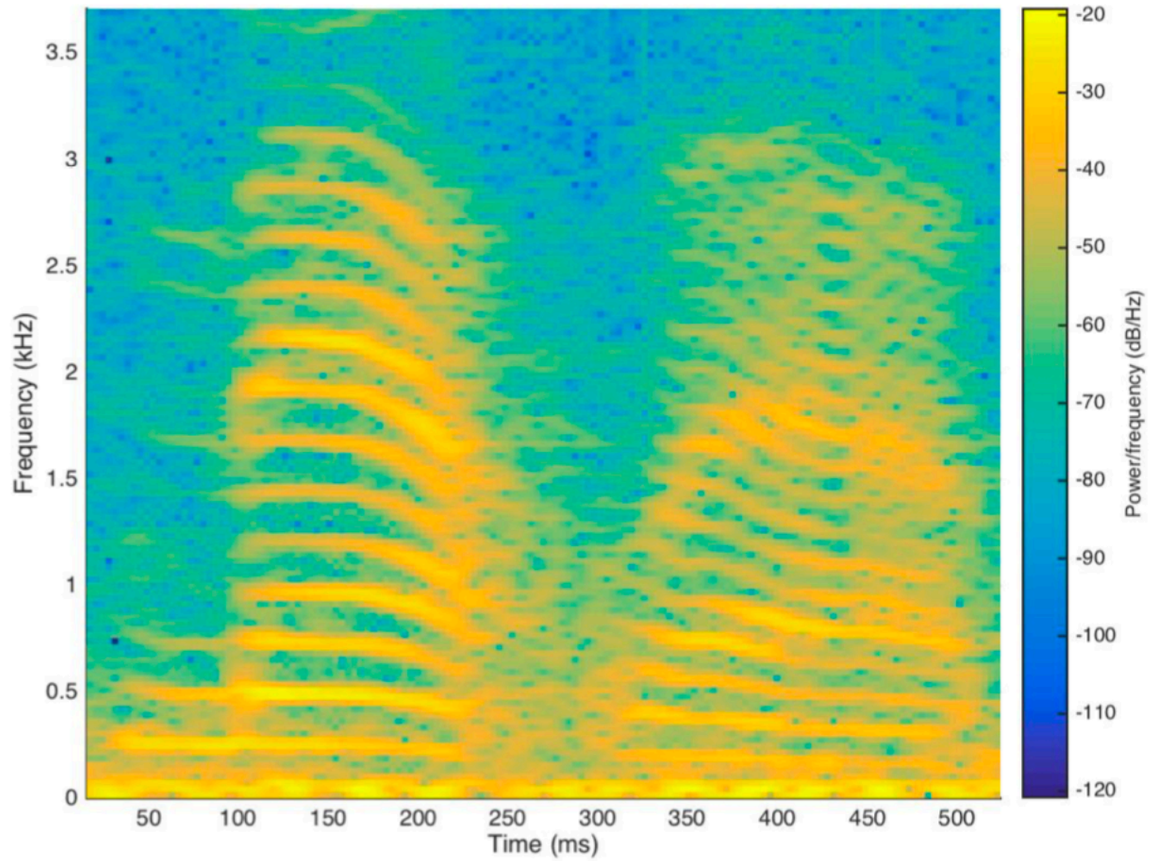
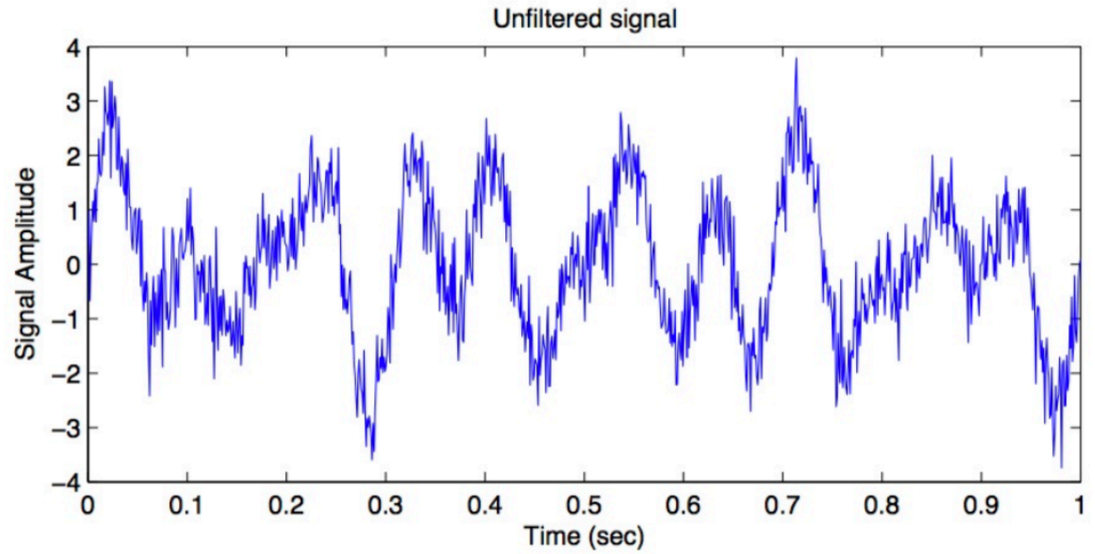
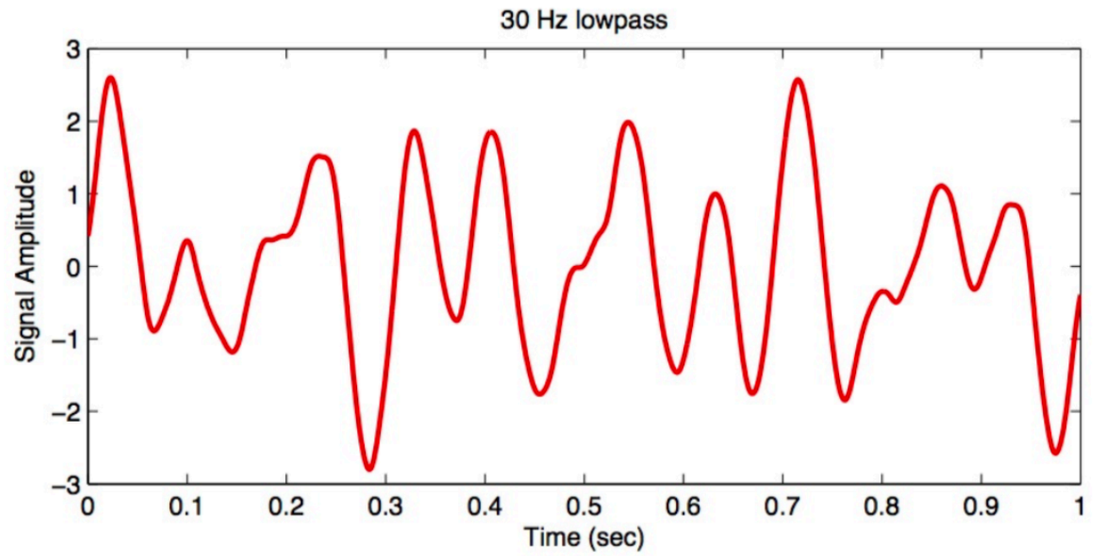


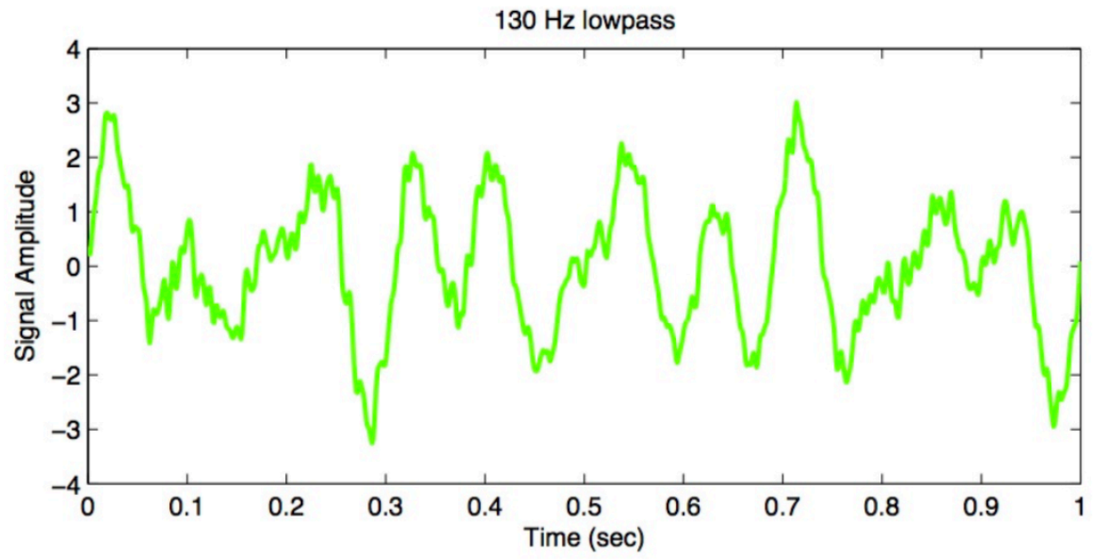
Figure 7: Spectrogram of the sound "MATLAB".

Filtering

- lowpass
- highpass
- bandpass
- bandstop

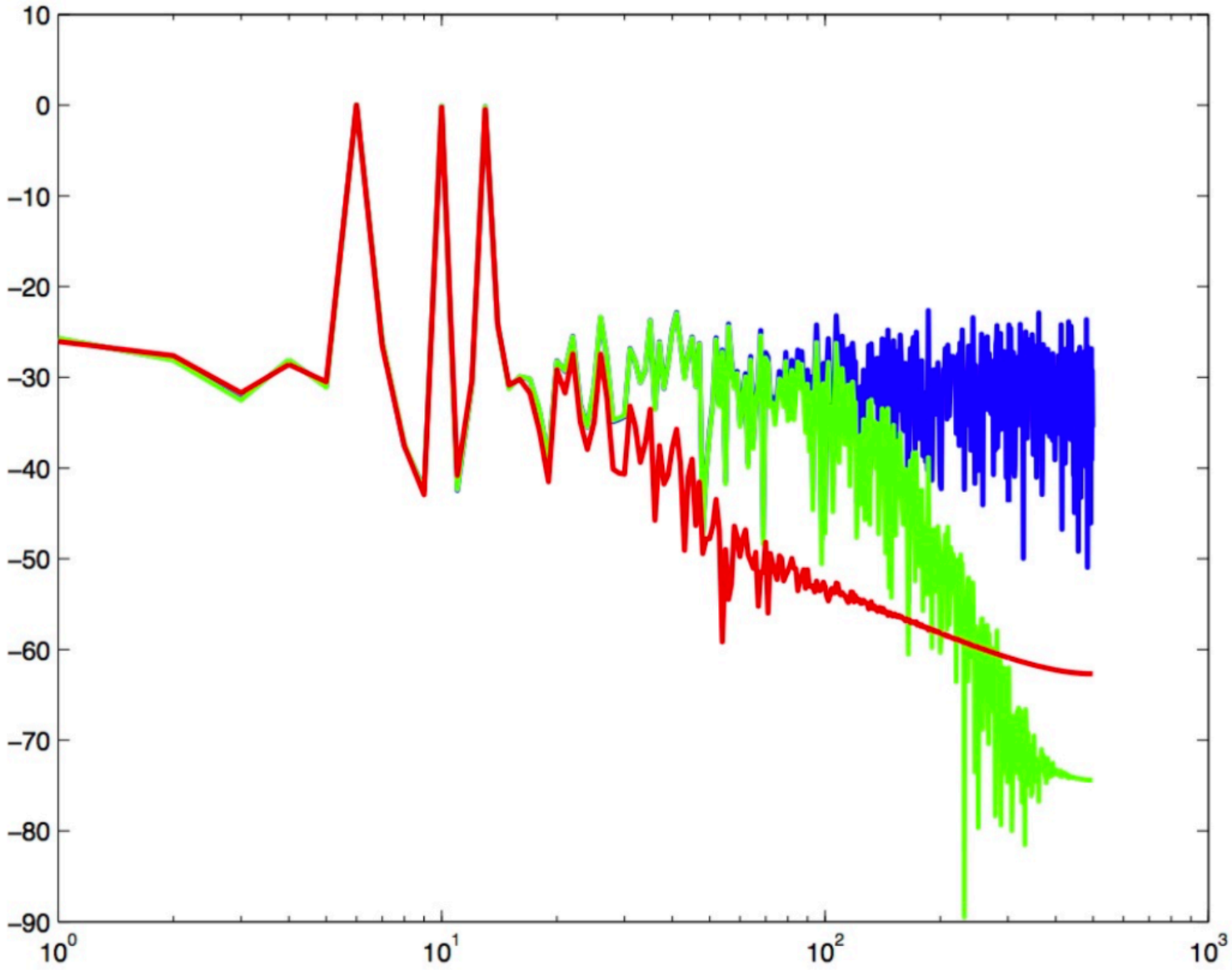






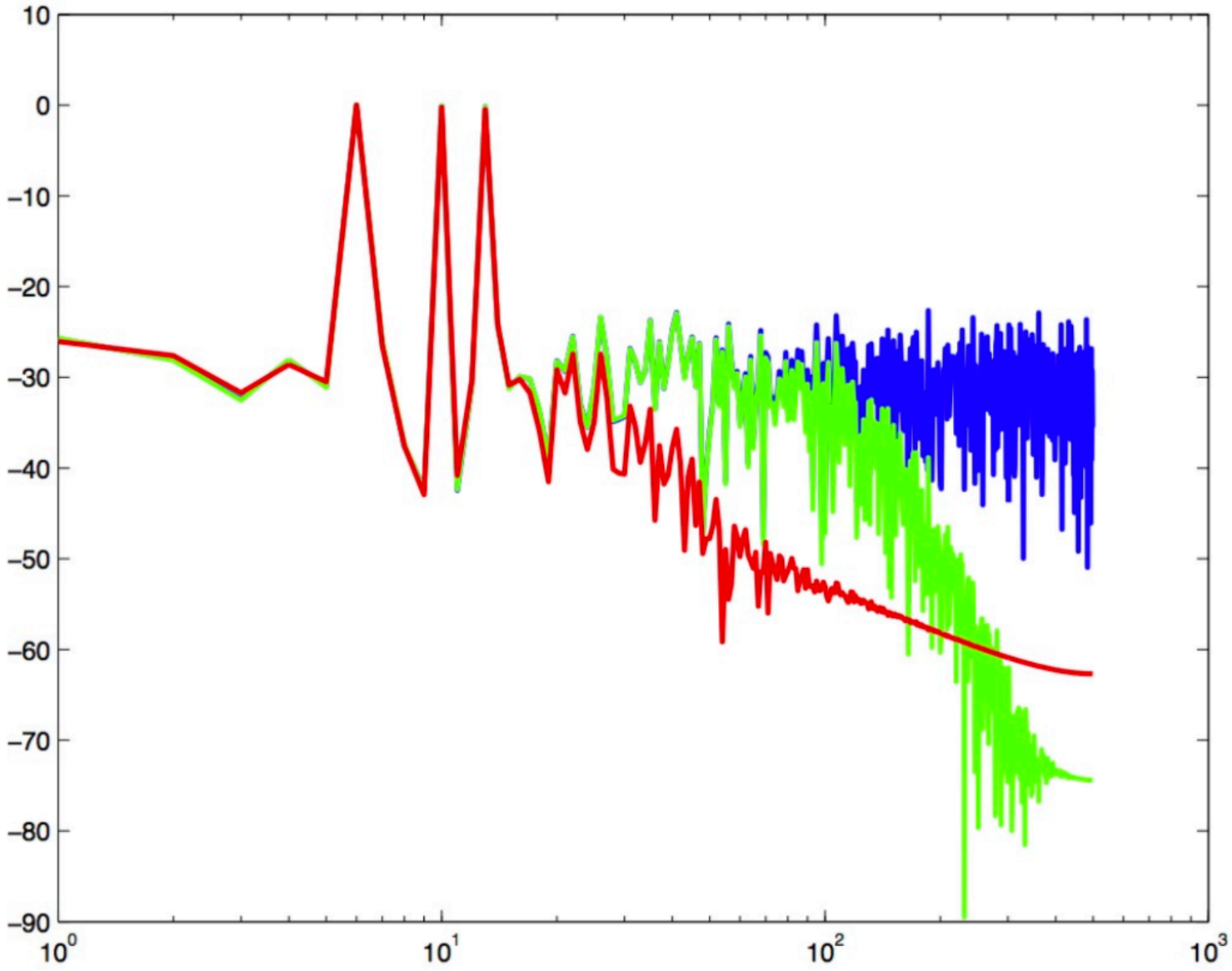
Filtering

- lowpass
- highpass
- bandpass
- bandstop

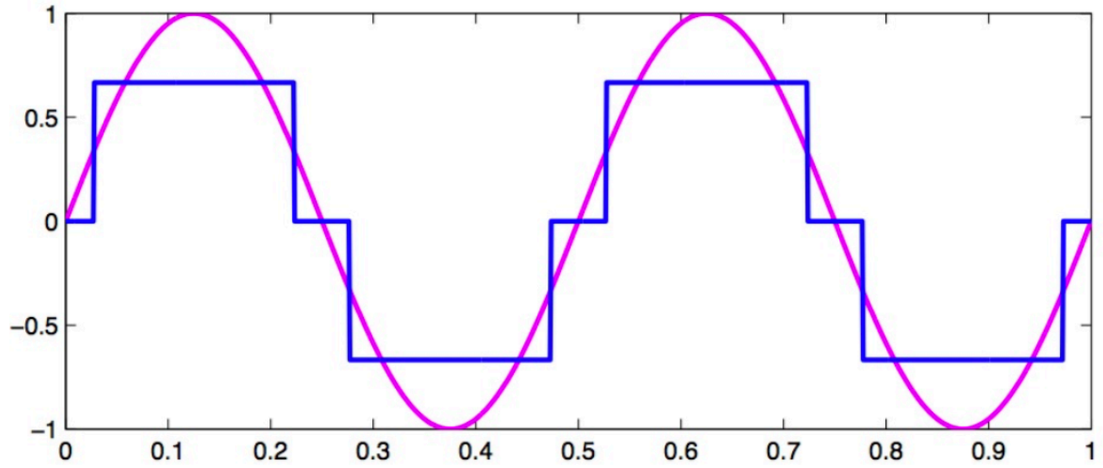


Filtering

- cutoff or corner frequency
- pass band, stop band
- rolloff



Quantization



- like a sampling rate but not over time, but over the range of the input signal

- usually expressed in number of **bits** over input range in **Volts**

- 12-bit vs 16-bit A/D board